Bending moment capacity of rectangular mortise and tenon furniture joints

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Abstract
This study investigated the effects of wood species, adhesive type, rail width, tenon depth, and tenon length on bending strength and flexibility of mortise and tenon T-type end to side grain joints. Sugar maple, red oak, walnut, and tulip poplar were utilized in constructing the joint specimens. Specimens were assembled with four types of adhesives. Nineteen groups of mortise and tenon joints with five replications each were tested in bending. Results of the tests indicated that a mortise and tenon joint becomes stiffer as either tenon length or tenon depth is increased. The results also indicate that tenon depth has a more significant effect on joint flexibility than tenon length. Furthermore, the presence of a shoulder on the rail member of a mortise and tenon joint substantially contributes to the stiffness of the joint. An empirically derived expression was developed to predict average ultimate bending moment capacity. Semi-rigid connection factors were also determined.

The rational design of furniture constructed with mortise and tenon joints dictates that expressions be available for estimating the bending moment capacity of these joints. Previous studies have defined several of the factors that affect their strength. Milham (1949), for example, demonstrated that the strongest joints are obtained when a close tolerance is maintained between the tenon and mortise and, furthermore, that the shoulders have a pronounced effect on the bending moment capacity of the joint. Dupont (1963) showed that optimum joint strength was obtained when glue was applied to both the tenon and sides of the mortise. Willard (1966a, 1966b, 1967) showed that joints with compressed tenons were no stronger than conventional joints. Sparkes (1968) concluded that square-end and round-end mortise and tenons were equally effective but that a square-end tenon fitting into a round-end mortise produced joints that were 15 percent weaker than either of the other two. He also found that as tenon depth and length increased the strength of the joints increased correspondingly.

Kamenicky (1975) investigated the flexibility of spruce and beech mortise and tenon joints and determined flexibility factors to characterize their behavior. He concluded that tenon depth greatly influences joint flexibility and that small joints may be treated as though they were hinged. Kamenicky (1977) also investigated the in-plane, out-of-plane, and torsional moment capacities of mortise and tenon joints and found that for a specified tenon length, bending moment capacity is closely related to tenon depth (Fig. 1). He also found that torsional moment capacity is independent of tenon length.

Miyajima and Sato (1977) compared the strength of L-shaped dowel, mortise and tenon, and finger joints and showed that finger joints had the highest bending moment capacity. Subsequently, Ishii and Miyajima (1981) investigated the effect of tenon length and tenon depth on bending moment capacity and found that near linear relationships existed between these factors and moment capacity.

Watanuki et al. (1981) compared the bending moment capacity of mortise and tenon joints with dowel and finger joints and made recommendations for the appropriate use of each joint. Paulenkova (1984) compared the strength of dowel and mortise and tenon joints and found that for comparable joint sizes, mortise and tenon joints had a decided strength advantage. Yang and Lin (1986a, b) investigated the effect of tenon fit and the effect of joint geometry factors on the strength of blind mortise and tenon joints and found that maximum bending mo-
ment capacities were obtained with tenon/mortise fits of −0.008 to +0.012 inches. They also found that maximum bending moments were obtained with a tenon to rail thickness ratio of one-half rather than the rule of thumb value of one-third.

Finally, Mihailescu (2001) compared the stresses and deflections obtained in both square and round mortise and tenon joints using finite element methods and found no significant differences.

The study reported in this paper was undertaken to obtain additional information about the bending moment capacity of mortise and tenon joints. The specific purpose of the study was to develop a predictive expression for mortise and tenon joints that would take into account wood species, adhesives, and joint geometry specifically, tenon depth, tenon shoulder width, and tenon length.

**Test specimens**

Typical configurations of the T-type end to side grain mortise and tenon joints (Fig. 1) used in this study are shown in **Figure 2**. All of the material measured 1 inch thick. Two rail widths, namely, 2 inches and 3 inches, were used to create 11 possible mortise and tenon configurations. A listing of the various mortise and tenon dimensions, wood species, and adhesives used in each experiment are given in **Table 1**.

Mortises were machined into the posts with a 3/8-inch hollow chisel bit mounted in a drill press. The depth of the mortise was 1/8 inch deeper than the tenon length. The tenons were machined on a table saw with a special jig to ensure that all of the cuts were made parallel to the sides of the rail and perpendicular to the surface of the table saw.

Tenon thickness was controlled by means of a spacer ring placed between two matched circular saw blades so that the final tenon thickness of the rail member when cut with this saw was approximately 0.373 inches, which produced a nominal mortise-tenon clearance of 0.001 to 0.002 inch.

The adhesives used in this study included a 65 percent solids polyvinyl acetate (PVA), urea-formaldehyde (UF), phenol-resorcinol (PR), and an animal glue. Adhesives were applied liberally to all faces of the tenon and to the sides and bottom of the mortise. Wax paper was slipped over the end of the tenon before assembly to prevent the shoulder of the rail from bonding to the wall of the post. Prior to, and following assembly, all

**Figure 1.** — Nomenclature of a blind mortise and tenon joint.

**Figure 2.** — Geometries of the various mortise and tenon joints tested in this study. The thickness of all tenons was 3/8 inch. The depth and length of the tenon are given below each specimen configuration.
of the specimens were stored in a climate controlled room that maintained an equilibrium moisture content of 6.5 to 7 percent in the wood.

Description of tests

In the first set of tests, 25 specimens were evaluated to determine the effect of tenon length on ultimate bending moment capacity and joint flexibility. All variables except tenon length were held constant. Tenon length varied from 0.5 to 2 inches. In the second set of tests, 30 specimens were evaluated to determine the effect of tenon depth on bending moment capacity and joint flexibility. Tenon depth varied from 0.5 to 3 inches while rail width and tenon length were held constant at 3 inches and 1 inch, respectively. In the third set of tests, 20 specimens were evaluated to determine the effect of wood species on moment capacity of the joint. Rail width and tenon depth were 2 inches while tenon length was 1 inch. Wood species included sugar maple (*Acer saccharum*), red oak (*Quercus rubra*), black walnut (*Juglans nigra*), and yellow-poplar (*Liriodendron tulipifera*). Finally, in the fourth set of tests, 20 specimens were evaluated to determine the effect of the four types of adhesives on moment capacity. Again, rail width, tenon depth, and tenon length were held constant at 2 inches, 2 inches, and 1 inch, respectively.

All tests were carried out on a Riehle universal testing machine. The post of each specimen was bolted to a testing jig and mounted in the testing machine (Fig. 3) in such a way that the load head of the testing machine contacted the rail at a distance of 12 inches from the face of the post so that the bending moment acting on the joint was equal to 12 times the applied load. Measurements of joint flexibility were obtained by means of a dial gage clamped to the top edge of the rail. The horizontal axis of the dial gage was located at a point 2 inches above the upper edge of the rail (Fig. 3). Dial gage readings were taken at regular intervals as the specimens were loaded.

Results

Bending moment capacity

Average ultimate bending moment values for the joints are given in Table 1. In general, joints with tenon lengths of 0.5 to 1 inch failed because of splitting of the post, whereas joints with tenon lengths of 1.5 to 2 inches failed because of fracture of the tenon. It is interesting to consider the maximum bending moment capacity that could be expected. The ultimate bending moment capacity of a 3/8-inch-thick by 2-inch-deep sugar maple tenon considered as a beam is given by the expression:

\[ \sigma = \frac{6M}{b \times d^2} \]  

where:

- \( \sigma \) is the stress in psi,
- \( M \) is the bending moment in in-lb,
- \( b \) is the width of the tenon,
- \( d \) is the depth of the tenon.

Table 1. — Comparison of average test results obtained in this study with values obtained with Equation [1].

<table>
<thead>
<tr>
<th>Rail width</th>
<th>Tenon depth</th>
<th>Tenon length</th>
<th>Adhesive</th>
<th>Wood species</th>
<th>Actual strength</th>
<th>Predicted strength</th>
<th>Difference</th>
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<td>2.00</td>
<td>0.50</td>
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<td>1.00</td>
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<td>3,470</td>
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<td>2.00</td>
<td>2.00</td>
<td>1.00</td>
<td>AN</td>
<td>Sugar maple</td>
<td>3,257</td>
<td>3,303</td>
<td>1.4</td>
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</table>

* UF is urea-formaldehyde; PR is phenol-resorcinol; PVA is polyvinyl acetate; and AN is animal adhesive.
\[ \sigma = \text{modulus of rupture (MOR) of the sugar maple at 7 percent moisture content (MC), 19,293 psi (USDA 1999),} \]

\[ M = \text{ultimate bending moment capacity of a 3/8-inch-thick by 2-inch-deep tenon (in-lb), and} \]

\[ b \text{ and } d = \text{the thickness and depth of a tenon, respectively (in).} \]

Rearranging terms and solving gives:

\[ M = 19,293 \times \left(\frac{3/8 \times 2^2}{6}\right) = 4,823 \text{ in-lb} \]

Thus, the joints with 1-1/2- and 2-inch-long tenons developed the maximum expected bending moment capacity of the tenons, whereas the joints with shorter tenons did not.

The post failures were attributed to fracture of the mortise wall in tension perpendicular to the grain. If the thickness of the post had been increased, post failure might have been eliminated and the strength of the joint increased.

The functional relationship between tenon length and bending moment capacity was evaluated by means of the regression expression:

\[ M = a_0 L^{a_1} \]

where:

\( M \) = the ultimate bending moment capacity of a joint (in-lb),

\( L \) = the length of a tenon (in), and

\( a_0 \) and \( a_1 \) = regression coefficients.

When this expression was fitted to the results of the first set of tests, the expression \( M = 3,379^{0.797} \) resulted with \( r^2 = 98.9 \) percent. This result indicates that there is a near linear relationship between tenon length and bending moment capacity. Specifically, the greatest difference that arises between the use of the above curvilinear expression and a straight line relationship is 3.7 percent, which occurs with the 0.5-inch tenon length.

A distribution of stress in the tenon and rail similar to that shown in Figure 4 is assumed to reflect the effect of tenon depth and shoulder width on bending moment capacity. The regression expression:

\[ M = a_0 [a_1 (W-D)/2 + a_2 D] \]

was used to evaluate these effects, where:

\( W \) = rail width (in),

\( D \) = tenon depth (in), and

\( a_0 \) to \( a_7 \) = regression coefficients.

When this expression was fitted to the results of the second set of tests, the expression:

\[ M = 3,387 \times [0.25 \times (W-D)/2 + 0.453 \times D] \]

resulted with \( r^2 = 99.3 \) percent. This result clearly shows the substantial effect that shoulders have on the bending moment capacities of the joints.

Finally, an expression of the form:

\[ M = a_0 [a_1 (W-D)/2 + a_2 D] L^{a_3} \times S \times \left(1 + a_4\right) \left(1 + a_5\right) \left(1 + a_6\right) \left(1 + a_7\right) \]

was fitted to the complete set of test results where:

\( S \) = the shear strength of the wood (psi) and

\( a_0 \) to \( a_7 \) = regression coefficients where:

\( a_4 \) relates to the presence or absence of PR,

\( a_5 \) to UF,

\( a_6 \) to PVA, and

\( a_7 \) to animal glue.

Results of this analysis (\( r^2 = 97.4 \)) were then simplified to yield the expression:

\[ M = a \times \left[0.25 \times (W-D) + 0.78 \times D\right] \times L^{0.8} \times S \]

where:

\( a = 1.00 \) for joints constructed with 65 percent PVA,

0.74 for PF,

0.79 for animal, and

0.83 for UF adhesive.

The ability of the above expression to estimate the ultimate bending strength of the mortise and tenon joints tested is shown in Table 1 where calculated strength values are compared to the actual test results. With the exception of the specimens constructed of yellow-poplar, the predicted and actual values agree closely. For practical design purposes, this regression expression – which illustrates “shoulder effect” – can be further simplified to the form:

\[ M = a \times \left[0.25 \times W + 0.50 \times D\right] \times L^{0.8} \times S \]

with little loss in accuracy. This expression, for example, on average, predicts the values obtained by Sparkes (1968) within 1 percent.

Stiffness

The rotation that occurs in semi-rigid joints is commonly described by the semi-rigid connection factor \( Z \), which, in turn, is defined by the expression:

\[ Z = \phi / M \]

where:

\( \phi \) = the angle change resulting from the semi-rigid behavior of the connection, radians, and

\( M \) = the bending moment acting on the joint (in-lb) (Lothers 1960).
Semi-rigid connection behavior was evaluated through the use of the test set up shown in Figure 3.

Values of the connection factors were determined by means of the expression:

\[ Z = \frac{y_1 + y_2}{2 + 2 + w} \times \frac{1}{M} \]

where:
- \( y_1 \) and \( y_2 \) = the absolute values of the dial gage deflections (in),
- \( 2 + 2 + w \) = the distance from the centerline of the top gage to the centerline of the bottom gage (in), and
- \( M \) = the bending moment acting on the joint (in-lb).

Average \( Z \)-values for the various joint geometries are given in Table 2. These results indicate that the joints become increasingly stiffer as rail width, tenon depth, tenon length, or any combination of these is increased. The results also indicate that tenon depth has a more substantial effect on joint flexibility than tenon length. Furthermore, the shoulders on the tenon member substantially affect the stiffness of the joint; for example, the joints constructed with a 2-inch-deep by 1-inch-long tenon in the first set of specimens (\( W = 2 \) in) had a \( Z \)-value of \( 5.185 \times 10^{-6} \) radians/in-lb, whereas the joints constructed with comparable tenons in the second set (\( W = 3 \) in) had a \( Z \)-value of \( 4.321 \times 10^{-6} \) radians/in-lb. Thus, the specimens that contained shoulders were 20 percent stiffer than specimens with the same tenon size without a shoulder.

**Conclusion**

The mortise and tenon joints examined in this study performed in a predictable manner so that the several variables affecting the strength of the joints could be evaluated individually and then incorporated into a comprehensive expression that reflects the functional relationship of these variables to the ultimate performance of the joint.

The empirically derived predictive expression – which predicts the ultimate bending moment capacity of mortise and tenon joints – provides furniture designers with a design tool that makes it possible to predict the strength of joints constructed of essentially any species. It must be remembered, however, that the predictive formula developed in this study is based on the ultimate bending strength of mortise and tenon joints that were constructed in the laboratory under carefully controlled conditions and predicts only ultimate strength values. In practice, joints are not manufactured under these ideal conditions and a reduction in strength must be expected.

Finally, the semi-rigid connection factors determined for the joints make it possible for product engineers to conduct more exact analyses of furniture constructions that utilize these joints.

**Literature cited**


