Rigidity of furniture cases with various joint constructions

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Abstract
Although expressions have been developed to predict the deflection characteristics of five-sided furniture cases, these expressions do not take into account the stiffness of the joints. This study was carried out to determine the effect of joint rigidity on case stiffness. Results indicate that joints do have a significant effect, and manufacturers may want to use joints that provide the greatest stiffness in their constructions.

Cases are one of the most important types of furniture produced, yet relatively little research has been done on the construction of furniture cases. And few papers have been published concerning their rational analysis.

Kotas (12,13) carried out the first known study of the structural characteristics of case furniture; the results of this research were later incorporated into a small design manual (14). Eckelman (1,2) subsequently developed a method of analysis for cases based on the interrelated deflections of the various corners and the stiffnesses of the individual panels. Ganowicz and Rogoziński (10) applied the principles of internal work to the analysis of case furniture, and Ganowicz et al. (7) extended this work. Ganowicz and Kwiatkowski (9) subsequently carried out tests on a case and experimentally evaluated the forces acting at the corners. Hata (11) analyzed the effect of the back panel and the depth of a case on stiffness. Eckelman and Resheidat (6) also evaluated case stiffness and presented formulas for determining the forces acting at each corner of the panels in a case.

All of these studies dealt with the evaluation of open-faced five-sided cases; none dealt with the treatment of the shelves and partitions in a case. Subsequently, Eckelman and Rabiej (5) evaluated these effects by means of a finite element type of analysis (3) and presented a simple method for evaluating the stiffness of cases containing these parts based on classical methods which continued and extended their previous method of analysis.

The methods of analysis developed in these studies are based on certain simplifying assumptions: 1) the panels do not deform in their own plane, and 2) the edge connections between panels do not transfer bending force from one panel to another. In practice, the first assumption is likely fulfilled in cases constructed of panels; the second assumption is not fulfilled, however, because the edge joints in almost every case construction possess some degree of rigidity. In general, the second assumption would be satisfied only if a hinge joint were used between adjacent panels, which could transfer shear and axial forces but not bending forces. An analysis based on the second assumption would predict deflections with reasonable closeness, however, provided the panels comprising the case were thin enough that the out-of-plane bending forces carried by them were negligible. In all practical constructions, the joints will be at least semirigid (2,15). Therefore, the cases should be stiffer than predicted because internal resisting bending forces, however small, will be developed in the panels. An indication of the stiffening effect of the joints is provided in the work of Ganowicz and Kwiatkowski (9). They found that a case constructed with special hinged joints and a screwed-on back was 8 percent stiffer than theory predicted. They also found that a case constructed with dowel joints and a screwed-on back was 31 percent stiffer than predicted. These results indicate that the stiffness of a case may be significantly affected by the type of joint used. It is important, therefore, that the effects of various types of
joint constructions be investigated in order to obtain realistic estimates of their effect on case deflection. Such research is necessary to obtain both realistic estimates of deflection for design purposes and to determine those joints which most effectively stiffen a case. The objective of this study, accordingly, was to determine the extent to which certain joints stiffen a case, and to broaden and extend the overall body of knowledge of case performance behavior.

Experimental methods

Four cases constructed with three types of joints were evaluated during this study: one case with hinge joints, one case with deliberately stiffened joints, and two cases with dowel joints. The general configuration of the cases is shown in Figure 1.

All cases were constructed of particleboard which had a density of 47.5 pcf based on oven-dry weight and volume at 5.1 percent moisture content (MC). Modulus of rupture (MOR) of the board was 3,392 psi; modulus of elasticity (MOE) was 655,000 psi. The top, bottom, and side panels of all cases were 3/4 inch thick; the rear panels were 3/8 inch thick. The height, width, and depth of each case measured 60, 30, and 13-7/8 inches, respectively.

The details of construction of the various joints used in construction of the cases are given in Figures 2a through 2f. In the case with hinged joints, (Fig. 2a), the panels were joined together with high quality butt hinges. Two hinges (one each, front and back) were used to attach adjacent members together, except for the rear panel. The rear panel was attached to the hinges at each rear corner by means of a metal strap with a hole drilled through it at one end, perpendicular to the broad face (Fig. 2b). The rear panel was then attached to the hinge joint at each corner by means of the hinge pin, which first passed through the slightly oversize hole in the end of the strap, and then into the hinge itself. Axial forces could thus be transmitted to the hinge pin by the rear panel, in each of the three coordinate directions, but bending forces could not be transmitted.

Brackets constructed of 1-1/4 by 1-1/4 by 1/4-inch thick angle iron, which were 13-1/2 inches long (Fig. 2c), were used to reinforce the joints in the second case. Each angle was bolted to each panel by four 5/16-inch diameter bolts, which were spaced 3-7/8 inches apart. The rear panel was screwed and glued to the rear edges of the top, bottom, and side panels in this construction (Fig. 2d).

The sides, top, and bottom of the first case constructed with dowel joints (Fig. 2e) were connected together with four 3/8-inch diameter by 1-1/2-inch long multigrooved yellow birch dowels, at each common
edge. Depth of embedment was 1/2 inch in the face of the side panels, and 1 inch in the edges of the top and bottom panels. Dowel spacing was 3-7/8 inches. The left and right sides overlapped the ends of the top and bottom. All joints were double-glued with a PVA adhesive (60% solids) furnished by Franklin Chemical Industries. The rear panel was screwed and glued to the top, bottom, and sides.

Construction of the second case with dowel joints (Fig. 2f) differed from the first in that the dowels were embedded 1 inch in the edges of the side panels and 1/2 inch in the face of the top and bottom panels (i.e., the top and bottom panels overlapped the ends of the right and left sides). Otherwise, construction was identical to that of the previous case.

Testing procedures

The stiffness of each panel used in the case was determined by means of the apparatus shown in Figure 3. The stiffness of each panel was then computed by means of the expression:

\[ \frac{f(i)}{y(i)} = \frac{f \times m}{L_2} \frac{n}{(y_1 + y_2) L_2} \frac{L}{L_1} \]  

where:

- \( f(i)/y(i) \) = stiffness of the panel
- \( L \) = distance between the clamps on the ends of the panels
- \( m \) = torsional moment arm
- \( n \) = distance between the two dial gages
- \( L_1 \) = distance between supports measured along the length of the panel (also equals the distance from the supports at the one end of the panel to the dial gages at the other end of the panel)
- \( L_2 \) = width of the panel
- \( y_1, y_2 \) = deflections of the dial gages (absolute values).

The term \( L/L_1 \) corrects for that portion of the panel which was clamped at each end; the term \( n/L_2 \) relates the observed deflection values to the width of the plate; and finally, the term \( f \times m/L_2 \) relates the applied tor-
Figure 3. — Method used for testing panel stiffness.

Figure 4. — Diagram showing two deformations of interest which arise from loading an unsupported corner of a case.

Figure 5. — Diagram showing placement of dial gages to measure deflections during tests.

TABLE 1. — Stiffnesses of the panels used in the various cases. *

<table>
<thead>
<tr>
<th>Case type</th>
<th>Bottom</th>
<th>Top</th>
<th>Left side</th>
<th>Right side</th>
<th>Rear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge joint</td>
<td>102.7</td>
<td>100.4</td>
<td>43.4</td>
<td>45.4</td>
<td>3.3</td>
</tr>
<tr>
<td>Angle joint</td>
<td>90.3</td>
<td>87.5</td>
<td>48.0</td>
<td>45.8</td>
<td>3.3</td>
</tr>
<tr>
<td>Dowel joint No. 1</td>
<td>92.9</td>
<td>97.5</td>
<td>43.5</td>
<td>43.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Dowel joint No. 2</td>
<td>92.5</td>
<td>95.5</td>
<td>44.3</td>
<td>44.6</td>
<td>3.2</td>
</tr>
</tbody>
</table>

*All values are given in units of pounds/inch.

The values of the stiffnesses of the individual case panels as determined by Equation [1] are given in Table 1. Since the top, bottom, and sides were constructed of the same material, the moduli of rigidity of the panels would be expected, on the average, to be the same. A comparison of the ratio of the lengths of the top and bottom to the sides, with the stiffness ratios for the panels as given in Table 1, indicated that the ratios determined from measured stiffness values exceeded the length ratios by only 5 percent. This result indicates that the test method did, in fact, provide consistent results.

Theory predicts that the upper rear left-hand corner of the case (joint 7 of Fig. 5) should not deflect during loading. In practice, deflections in gage directions 10 and 11 (Fig. 5) could not be prevented by ordinary means. To eliminate the effects of these deflections from the test results, their contribution to total measured deflections were subtracted at each of the other gage points.

To provide the best estimates of case stiffness, regression curves were then fitted to the resultant force versus deflection data in the 1, 5, 6 gage directions. These actions were selected for evaluation because they are the largest which occur, and also because all other deflections can readily be related to them (Fig. 4). The stiffness values for each case at these three points are given in Table 2.
TABLE 2. — Adjusted deflections of the case, $Y_1$, $Y_2$, and $Y_3$.

<table>
<thead>
<tr>
<th>Type of joint</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge joint</td>
<td>0.006628</td>
<td>0.003075</td>
<td>0.001524</td>
</tr>
<tr>
<td>Angle joint 1</td>
<td>0.006598</td>
<td>0.002203</td>
<td>0.001155</td>
</tr>
<tr>
<td>Dowel joint 1</td>
<td>0.006207</td>
<td>0.002748</td>
<td>0.001411</td>
</tr>
<tr>
<td>Dowel joint 2</td>
<td>0.006690</td>
<td>0.003058</td>
<td>0.001544</td>
</tr>
</tbody>
</table>

*Deflections were measured at points 1, 4, and 8.

Discussion

The theoretical deflection of each case may be calculated by means of the following expression developed by Eckelman (1):

$$Y_3 = \frac{F}{f(b) + f(t) + b \frac{f(sl)}{y(b)} + \frac{f(sl)}{y(sl)} + \frac{f(sr)}{y(sr)} + \frac{b^2 f(r)}{c^2 y(r)}}$$

In this equation, $a$, $b$, and $c$ refer to the width, height, and depth of the case, respectively, (in.); $f(b)/y(b)$, $f(t)/y(t)$, $f(sl)/y(sl)$, $f(sr)/y(sr)$, and $s(r)/y(r)$ refer to the stiffnesses of the bottom, top, left side, right side, and rear panels, respectively, (lb/in.); and $F$ refers to the magnitude of the external load applied to the rear corner of the case in a vertical direction. In these calculations, it is assumed that the plates do not deform in their own plane, and that the edge joints do not transfer bending force from one panel to another. Once $Y_3$ is known, $Y_1$ and $Y_2$ may be calculated by means of the following geometrical relationships (Fig. 4):

$$Y_1 = \frac{(b/a)}{Y_3}$$
$$Y_2 = \frac{(b/c)}{Y_3}$$

If the appropriate panel stiffnesses are substituted into Equation [2], $Y_3$ is obtained for each case. Similarly, if the corresponding values of $Y_1$ and $Y_2$ are computed, the stiffness values given in Table 3 are obtained for the various cases. The differences between adjusted and predicted deflections may then be expressed as a percentage of predicted deflections. These percentages are shown in Table 4 for each of the various cases; in addition, the average of $Y_1$, $Y_2$, and $Y_3$ is given for each case type.

As can be seen in Table 4, deflection of the hinge-jointed case was 5 percent less than predicted. This result represents close agreement between theory and practice, and indicates that the assumptions made concerning the joints in the theoretical analysis were nearly satisfied in the test case.

Deflection of the case with the joints reinforced with angle iron brackets was 30 percent less than predicted. This result indicates that for panels of the geometry used in this study, considerable stiffening of the case can be obtained by increasing the rigidity of the joints. In this study, the joints were reinforced with the angle iron brackets; presumably, comparable results could be achieved in practice through the use of glued-in-place wooden rails.

The stiffnesses of the two dowel-jointed cases were 7 percent and 10 percent less, respectively, than the values predicted for a hinge-jointed case. This indicates that these joints are relatively flexible in their structural action, but they still contribute a small amount to the stiffness of the case.

TABLE 3. — Theoretical deflections of the cases corresponding to $Y_1$, $Y_2$, and $Y_3$.

<table>
<thead>
<tr>
<th>Type of joint</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge joint</td>
<td>0.006576</td>
<td>0.003226</td>
<td>0.001613</td>
</tr>
<tr>
<td>Angle joint 1</td>
<td>0.007039</td>
<td>0.003255</td>
<td>0.001628</td>
</tr>
<tr>
<td>Dowel joint 1</td>
<td>0.007224</td>
<td>0.003341</td>
<td>0.001670</td>
</tr>
<tr>
<td>Dowel joint 2</td>
<td>0.007169</td>
<td>0.003316</td>
<td>0.001658</td>
</tr>
</tbody>
</table>

TABLE 4. — Percentage difference between predicted and adjusted deflection values $Y_1$, $Y_2$, $Y_3$.

<table>
<thead>
<tr>
<th>Type of joint</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hinge joint</td>
<td>4.99</td>
<td>4.68</td>
<td>5.53</td>
</tr>
<tr>
<td>Angle joint 1</td>
<td>27.65</td>
<td>32.52</td>
<td>29.0</td>
</tr>
<tr>
<td>Dowel joint 1</td>
<td>14.08</td>
<td>17.75</td>
<td>15.6</td>
</tr>
<tr>
<td>Dowel joint 2</td>
<td>6.68</td>
<td>7.78</td>
<td>6.90</td>
</tr>
</tbody>
</table>

Conclusions

Results of the tests indicate that the deflection characteristics of cases with hinge joints agree relatively closely with the results predicted for them by means of expressions that are based on models which assume no transfer of moment from panel to panel. Furthermore, the deflection characteristics of cases constructed with semirigid joints (such as dowels) did not differ greatly from predicted values. The deflection characteristics of cases with significantly stiffened joints, however, do differ considerably from predicted values. In this respect, it is worthwhile to consider the stiffening effects of corner blocks and glue blocks in practical case construction.

Literature cited
