In addition to bending, shear, and axial forces, the joints and members of a furniture frame must also be able to resist torsional forces which tend to rotate a member about its longitudinal axis. As a general rule, wooden parts are not loaded in torsion, but in furniture frame construction the esthetic design of the furniture itself often requires that members be loaded in this way. When an arm stump is attached to the side rail of a chair or sofa, for example, any side force applied to the top of the stump by the arm will be transmitted to the side rail as torsional force. Other, more practical reasons also exist. The seat springs used in upholstered furniture, for example, impose torsional forces upon the rails used in the seat frame. This is particularly true when sinusoidal loop-type springs are used which are stretched in place and thereby exert considerable end forces on the tops of the front and back rails, Figure 1. Several members in a furniture frame may also be inadvertently loaded in torsion if the piece of furniture is set on an uneven surface. If all four legs of a chair, for example, do not rest evenly upon the floor, torsional forces will be induced in several members of the frame.

In order to design furniture parts against torsional forces, methods or formulas must first be available which can be used to determine the stresses and deflections which are induced in them by these forces. Once these stresses and deflections can be calculated, the proper cross section for a part can be determined provided that the allowable design stress and the flexibility characteristics (modulus of rigidity) of the material are known. This information is required in the design process so that the inherent strength of the material will not be exceeded and deflections can be restricted to acceptable limits.

The calculations involved in using exact expressions to determine the value of the maximum torsional shear stress induced in a member of rectangular cross section are quite complex and the equations are accordingly largely impractical to use. As a result simpler expressions have been developed for general engineering use which give approximate, but reasonable results. One such commonly used formula (Fairman and Cutshall, 1953)

\[ S_{t} = \frac{(15a + 9b)}{5a^2b^2} T \]

where

- \( S_t \) = the maximum torsional shear stress induced in the member, psi
- \( T \) = the torsional twisting force acting on the member, pound-inches
- \( a \) = the length of long side of the cross section, inches
- \( b \) = the length of short side of the cross section, inches

Although this formula, as applied to wood, has no real basis in theoretical mechanics, it can be used for design

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Coefficients, ( k ), to be used in equation (2).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (a/b) )</td>
<td>Coefficient</td>
</tr>
<tr>
<td>1.0</td>
<td>0.141</td>
</tr>
<tr>
<td>1.5</td>
<td>0.196</td>
</tr>
<tr>
<td>2.0</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.281</td>
</tr>
<tr>
<td>6.0</td>
<td>0.299</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
</tr>
<tr>
<td>Infinite</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Twisting forces were applied to the specimens by means of the apparatus shown here. Steel plates were first clamped to each end of the specimen. One end of the specimen was then attached to the output shaft of the speed reducer unit which is shown at the right hand side of the picture. As the operator turned the crank attached to the input shaft of this unit, torsional forces were then exerted by the output shaft on the specimen. The other end of the specimen was attached to a crank arm which was attached to a shaft whose longitudinal axis was parallel to that of the specimen. As the specimen was twisted, the arm of the crank pressed down on a ring-shaped load cell which can be seen at the left side of the picture. Forces measured by the cell were recorded by the strip-chart recorder shown in the upper left portion of the picture. In this photograph a specimen has been placed in the machine and is ready for testing.

Figure 2b
At this point the specimen has been rather heavily loaded in torsion. Note the large amount of twist.

Twisting forces acting on a specimen were measured by means of the apparatus shown here. The arm of the crank pressed down on the steel ball as the specimen was loaded which in turn pressed down on the ring-shaped load cell. The distance from the longitudinal axis of the crankshaft to the vertical axis of the load cell was twelve inches so that the twisting force acting on a specimen was equal to 12 times the force registered by the load cell.

Figure 4
Twisting of the specimen was measured by means of the dial gages and extension arms shown in this prototype setup. As the specimen was twisted, two dial readings, \( d_1 \) and \( d_2 \), were taken. The unit rotation of the specimen was then determined as

\[
\theta = \frac{d_1 - d_2}{eL}
\]

where \( e \) = the distance from the longitudinal axis of the beam to the points of contact of the extension arms with the stems of the dial gage
\( L \) = distance between extension arms.

### TABLE 2

<table>
<thead>
<tr>
<th>Species</th>
<th>Nominal Thickness</th>
<th>Width</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>Total Number of Specimens in Each Class</th>
<th>Total Number of Specimens in All Widths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Poplar</td>
<td>1&quot;</td>
<td></td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Sweetgum</td>
<td>7/8&quot;</td>
<td></td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Black Walnut</td>
<td>1&quot;</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Sugar Maple</td>
<td>1&quot;</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Red Oak</td>
<td>1 1/8&quot;</td>
<td></td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Note: Each specimen was 24 3/4 inches long.
TABLE 3  Ultimate torsional shear stress results for the specimens tested.

<table>
<thead>
<tr>
<th>Wood Species</th>
<th>Mean M.C.%</th>
<th>No. of Samples</th>
<th>Av. Ultimate Stress (psi)</th>
<th>Standard Deviation (psi) of Ultimate Stress Values</th>
<th>Av. Shear Stress Ratios</th>
<th>Standard Deviation (psi) of Shear Stress Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Poplar</td>
<td>5.0</td>
<td>17</td>
<td>2504.4</td>
<td>291.5</td>
<td>1.748</td>
<td>0.202</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>5.5</td>
<td>15</td>
<td>3698.7</td>
<td>629.2</td>
<td>1.934</td>
<td>0.335</td>
</tr>
<tr>
<td>Black Walnut</td>
<td>4.2</td>
<td>2</td>
<td>3446.6</td>
<td>181.2</td>
<td>2.04</td>
<td>0.170</td>
</tr>
<tr>
<td>Red Oak</td>
<td>7.6</td>
<td>2</td>
<td>2521.0</td>
<td>117.1</td>
<td>1.25</td>
<td>0.042</td>
</tr>
<tr>
<td>Sugar Maple</td>
<td>5.0</td>
<td>5</td>
<td>5348.4</td>
<td>520.5</td>
<td>1.890</td>
<td>0.152</td>
</tr>
<tr>
<td>All hardwoods tested</td>
<td>5.0</td>
<td>41</td>
<td>3295.5</td>
<td>1.825</td>
<td>0.294</td>
<td></td>
</tr>
</tbody>
</table>

1 Shear Stress Ratios are defined as the calculated ultimate torsional shear stress divided by the value for shear stress parallel to the grain given in the Wood Handbook corrected to current moisture content.

TABLE 4  Wood Species Moduli of Rigidity

<table>
<thead>
<tr>
<th>Wood Species</th>
<th>Mean M.C.%</th>
<th>No. of Samples</th>
<th>Mean Moduli of Rigidity psi</th>
<th>Standard Deviation psi</th>
<th>Average Ratio E/G</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Poplar</td>
<td>5.0</td>
<td>17</td>
<td>99.193 x 10^3</td>
<td>25.488 x 10^3</td>
<td>18.76</td>
<td>4.46</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>5.5</td>
<td>15</td>
<td>142.269</td>
<td>26.387</td>
<td>13.15</td>
<td>2.52</td>
</tr>
<tr>
<td>Black Walnut</td>
<td>4.2</td>
<td>2</td>
<td>168.041</td>
<td>14.688</td>
<td>11.40</td>
<td>1.27</td>
</tr>
<tr>
<td>Red Oak</td>
<td>7.6</td>
<td>2</td>
<td>127.724</td>
<td>8.946</td>
<td>15.05</td>
<td>0.92</td>
</tr>
<tr>
<td>Sugar Maple</td>
<td>5.0</td>
<td>5</td>
<td>152.834</td>
<td>24.718</td>
<td>13.64</td>
<td>2.20</td>
</tr>
<tr>
<td>All hardwoods tested</td>
<td>5.0</td>
<td>41</td>
<td>126.244</td>
<td>33.988</td>
<td>15.55**</td>
<td>4.31</td>
</tr>
</tbody>
</table>

*Modulus of elasticity values, E, were based on values given in the Wood Handbook adjusted to current moisture content conditions.

**Average values for all hardwoods are based on individual test results.

TORSIONAL CONSIDERATIONS
FROM PAGE 44

purposes provided that a) it gives consistent results for members of all different sizes of rectangular cross section and, b) that a consistent relationship can be shown to exist between the torsional shear stresses predicted by it and published values for shear stress parallel to the grain.

Similarly, the torsional stiffness of a wooden member, which is referred to as its modulus of rigidity, may be approximated by means of the expression

\[ G = \frac{LT}{kab^3\Phi} \]  \hspace{1cm} (2)

where

\[ \Phi = \text{total twist of the member in radians} \]

\[ G = \text{torsional stiffness (mean modulus of rigidity of the member), psi} \]

\[ k = \text{coefficient whose values are given in Table 1 for various ratios of a/b} \]

\[ a = \text{the length of the long side of the cross section of the member, inches} \]

TABLE 5  Specimen Width-Inches

<table>
<thead>
<tr>
<th>Species</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>1-Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow Poplar</td>
<td>-</td>
<td>1.74</td>
<td>1.77</td>
<td>1.90</td>
<td>1.59</td>
<td>1.62</td>
<td></td>
</tr>
<tr>
<td>Sweetgum</td>
<td>-</td>
<td>1.89</td>
<td>1.82</td>
<td>1.98</td>
<td>-</td>
<td>2.21</td>
<td></td>
</tr>
<tr>
<td>Black Walnut</td>
<td>-</td>
<td>2.16</td>
<td>1.82</td>
<td>1.92</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Red Oak</td>
<td>-</td>
<td>-</td>
<td>2.15</td>
<td>1.25</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Sugar Maple</td>
<td>1.99</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.84</td>
<td></td>
</tr>
</tbody>
</table>

1 Shear Stress Ratios are defined as the calculated ultimate torsional shear stress divided by the value for shear stress parallel to the grain given in the Wood Handbook corrected to current moisture content.

Side frame of an upholstered chair or sofa. Any outward force applied to the arm or top of stump will cause a torsional force to act on the side rail.
tional structures are seldom loaded in torsion, the torsional strength and rigidity of most woods have not been extensively investigated. The results that are available (Markwardt and Wilson, 1935), however, indicate that torsional shear strengths of wood, as calculated by the commonly used engineering formulas, are about 1/3 greater than the values quoted for shear parallel to the grain in the Wood Handbook (Anon., 1955). Because of such considerations, the Wood Handbook states that for solid wood members, the allowable ultimate torsional shear stress may be taken as equal to the ultimate shear stress parallel to the grain for each species. The question that arises here is whether or not this same specification can be used to determine allowable torsional shear stresses for wood used in furniture. Since the considerations on which allowable stresses for wood used in conventional structures are based are somewhat different than those used in furniture construction, differences in allowable stress design specifications for these two types of structures must be expected.

The Wood Handbook also states that the torsional stiffness of a species (mean modulus of rigidity) of wood may be taken as 1/16 of its bending stiffness (modulus of elasticity parallel to the grain). This would be a desirable convention to use if it holds sufficiently well for furniture woods in the sizes commonly used in furniture construction. Published values (Wood Handbook, 1955) indicate the following ratios for the species listed:

<table>
<thead>
<tr>
<th>Species</th>
<th>Torsional Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ash</td>
<td>1/20.4</td>
</tr>
<tr>
<td>Yellow Birch</td>
<td>1/14.1</td>
</tr>
<tr>
<td>Yellow Poplar</td>
<td>1/13.9</td>
</tr>
<tr>
<td>Sweetgum</td>
<td>1/13.3</td>
</tr>
<tr>
<td>Walnut</td>
<td>1/13.6</td>
</tr>
</tbody>
</table>

These ratios obviously differ somewhat from the suggested value of 1/16, but, except for ash, the results are conservative. The question that exists here, however, is how much variation from this convention may be expected from piece to piece both within a wood species and among wood species. An exploratory study was carried out to obtain estimates of these characteristics for woods widely used in furniture construction and to evaluate torsional strength and rigidity requirements in a typical furniture construction.

Description of Test Equipment

All tests were carried out on the torsion testing machine which can be seen in Figures 2 through 4. Torsional forces were applied to each specimen by means of a crank which was attached to the output shaft of a speed reducer unit. Clamping plates were first attached to the end of each specimen, and the specimen was in turn bolted to the torque producing crank attached to the output shaft of the speed reducer. The input shaft of the
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Tabulated summaries of the tests are given in Tables 3 and 4. The mean shear stress ratio, Table 3, for all of the hardwoods tested was 1.525 with a standard deviation of 0.294. For the purposes of this paper “mean shear stress ratio” is defined as the ultimate torsional shear stress as calculated by equation (1) divided by the appropriate value for shear parallel to the grain given in the Wood Handbook after being corrected for moisture content. Yellow poplar, sweetgum, black walnut, and sugar maple were all found to have shear ratios which agree rather closely with the average value, but the shear ratio for red oak deviated considerably. Since only a limited number of red oak samples were tested, it is reasonable to question whether the results obtained are typical for the species. Further testing is needed to resolve this question since inclusion of the data for red oak in a
general design specification necessarily imposes a severe penalty on other species. On the other hand, the results obtained for a red oak may, in fact, be representative for the species, and other species which have similar stress ratios may also exist.

If we now calculate one-sided tolerance limits, TL, (Ostle, 1963) for the results of the tests we find that

\[ TL = 1.825 - 2.126 \times 0.294 = 1.20 \]

i.e., we can be 95 percent confident that at least 95 percent of all such material would have stress ratios equal to or greater than 1.20. This result lends support to the practice recommended by the Wood Handbook; namely, that the values given by the Wood Handbook for shear parallel to the grain be used as allowable design values for torsional stresses.

In Table 5 the stress ratios have been categorized according to specimen width as well as wood species in order to determine if any trends exist which indicate that the stress values calculated by equation (1) are related to the cross sectional geometry of the specimens. In the case of the yellow poplar, calculated stress ratios decrease slightly as the width of the specimen is increased. In the case of sweetgum the results are just the opposite. From a practical point of view, accordingly, there does not appear to be a well-defined trend in the data which would indicate that equation (1) is sensitive to specimen geometry.

The average modulus of rigidity of all of the hardwood specimens tested was 126,244 psi with a standard deviation of 33,998 psi, Table 4. For design purposes it is generally assumed, as was previously stated, that modulus of rigidity may be taken as 1/16th the modulus of elasticity (Anon., 1951) for the material. In these tests the average modulus of rigidity was 1/15.55 with a standard deviation of 4.31. This result agrees well with the suggested value, but the large standard deviation suggests that considerable differences must be expected. As an example, the ratio for yellow poplar is 1 in 18.76 whereas the ratio for sweetgum is only 1 in 13.15. Individual values, of course, may differ by even larger amounts. The lowest ratio noted, for instance, was 1 in 28.5, whereas the highest ratio observed was 1 in 9.3. In spite of these differences, which may be attributed to the natural variability of the wood, the results of these tests indicate that the most representative modulus ratio is 1 to 16.

Summarizing briefly, the results of these tests indicate that the practices suggested in the Wood Handbook for designing against torsion in conventional structures can also be used for designing against torsion in furniture. Specifically, it appears that the allowable ultimate torsional shear stress for a particular wood may be taken as the shear stress parallel to the grain for that species. Furthermore, the modulus of rigidity of a species may be taken as equal to 1/16 of its modulus of elasticity.

Torsional Requirements in Furniture

It is useful at this point to calculate the torsional strength and rigidity requirements of wood used in a representative furniture application. As was previously mentioned, one of the most common examples of a member loaded in torsion is the side rail on a chair or sofa which has an arm stumped attached to it at some point between the front and back rails. Any outward (or inward) force applied to the arm will be transmitted to the side rail as a tor-
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twisting force, or,
24/30 X 1200 = 960 pounds-inches,
whereas that portion of the side rail between the stump and the back rail
will carry only 6/30 X 1200, or, 240
pound-inches.
In designing the side rail, let us try a
cross section that measures 15/16 X 3
inches as a first approximation.
Substituting these values into equation
(1) we find the torsional stresses acting
on this member to be
\[ S_t = \frac{(15 \times 3 + 9 \times \frac{15}{16}) \times T}{5 \times (3)^2 \times (\frac{15}{16})^2} = 1.35 \times \frac{T}{\text{in}^3}. \]
The stress acting on the front portion
of the rail, which will obviously be the
most highly stressed region, will ac-
cordingly be
\[ S_t = 1.35 \times 960 = 1296 \text{ psi}. \]
Since the allowable design stress in
this case is 1600 psi, the member is
somewhat overdesigned. Let us,
therefore, try a rail with a cross sec-
tion of say 15/16 X 2 1/2 inches. A side
rail with this cross section would have
a torsional stress, \( S_t \), imposed on it of
\[ S_t = \frac{(15 \times 2\frac{1}{2} + 9 \times \frac{15}{16}) \times 960}{5 \times (2\frac{1}{2})^2 \times (\frac{15}{16})^2} = 1603 \text{ psi}. \]
In this case, the torsional stress acting
on the side rail would be essentially
equal to the allowable shear stress so
that we can conclude that a sweetgum
side rail with a cross section of
15/16 X 2 1/2 inches would satisfy the
stipulated design conditions.
It is also useful to calculate the
deflection of the top of the stump in an
outward direction under the loading
shown. Assuming small deflection
theory, the top of the stump will be
forced outward a distance, \( d \), which
can be determined from the relation-
ship
\[ d = \frac{\theta \cdot c}{k \cdot a^2 \cdot b^2 \cdot \frac{T}{G}} \]  
where \( c \) is the height of the stump as
previously defined and \( \theta \), the angle of
twist, is found by rearranging equation
(2) as follows:
\[ \theta = \frac{L}{k \cdot a \cdot b^2 \cdot \frac{T}{G}} \]  
For a rail with a cross section of 15/16
X 2 1/2 inches, the ratio of the long
to the short side is 2 1/2 divided by
15/16, or, 2.67. The coefficient, \( k \),
Corresponding to this ratio is obtained
from Table 1 and is equal to approxi-
mately 0.249 (use the ratio in the table
which is nearest to the calculated
value). Substituting the appropriate
values into equation (4) gives
\[ d = \frac{960 \times 12 \times 6}{0.249 \times 2.5 \times (15/16)^2 \times \frac{T}{G}} \]
\[ = \frac{960 \times 12 \times 6}{86.310} \]
\[ = \frac{102,500}{16} \]
The top of the stump would be ex-
pected, accordingly, to deflect out-
ward 86,310/102,500, or, 0.842 inches.
These calculations serve to point out
that wood is relatively flexible when
loaded in torsion and for this reason,
constructions which derive their rigid-
ity from members loaded in torsion
should generally be avoided.

Bibliography
   Mechanics of Materials. John Wiley and
   Sons, New York.
   Strength and Related Properties of Woods
   Grown in the United States. USDA Tech.
   Bul. No. 479.
5. Osile, R. 1963. Statistics in Research. The
   Iowa State University Press, Ames, Iowa.

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