Furniture Mechanics: The Analysis of Paneled Case And Carcass Furniture

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At first glance, an analysis of box, case, and carcass furniture appears almost precluded by the seemingly endless variety of structural types encompassed. A closer examination, however, reveals that drawers, cabinets, wardrobes, chests, and so on usually contain only two basic components - a frame and panels. In some instances the frame may be entirely lacking and the furniture made up entirely of panels, whereas in other furniture the panels may either be absent or structurally inoperative. The greater part of case and carcass furniture, however, contains both a frame and panels. These interact to some degree as a composite structure and are thus structurally indeterminate. An exact analysis of composite structures is quite complex as evidenced by the difficulties, for example, in analyzing a high rise office building (Benjamin, 1958) once walls, partitions, floors, and roof have been added to the basic skeletal framework. To attempt an exact analysis of frame and panel furniture at this time is not justified - first, because of the complexity of the analysis, and second and more important because so little is known about the elastic behavior of this type furniture. The number of assumptions that must be made to analyze internal forces would render the analysis meaningless when applied to the real structure. Certain types of frame and panel construction can be analyzed, however.

When a complete structural framework is clearly evident, it is often possible to treat the piece as a rigid frame (Eckelman, 1966a, b, c). Even though almost all case and carcass furniture has a thin plywood, masonite, or hardboard back securely attached to the frame, these boards often have little torsional resistance, and their only effect may be to prevent joint rotations and frame translation in the plane of the board. Also, these panels may work loose because of shrinking and swelling accompanying climatic changes or because of severe racking of the case so that the panel can become inoperative. In either event the effect of the back panel on the frame can readily be taken into account. Framed panels may also have little effect on the frame. Frequently, the edges of the central panel are waxed (Feirer, 1963) before assembly so that the panel has some freedom of movement. In furniture constructed of such panels the central panel by itself has little or no effect on the elastic behavior of the furniture although the stiles and rails surrounding it usually are an integral part of the case frame. In other
construction a frame may be entirely lacking so that the furniture may be treated as an assemblage of panels rather than a composite structure. Most drawers, for example, have no frame, and many cabinets and wardrobes consist mainly of panels with perhaps at most a light structural framework whose chief function is to hold the panels together. Often, bookcases also are made up only of solid panels. A standard institutional bookcase, 84 inches tall by 36 inches wide and 12 inches deep made up of 3/4 inch solid oak sides, top and bottom, with plywood back is a good example. It is this latter type of furniture with which this paper is concerned.

In contrast to a rigid frame structure, which derives its strength and rigidity from the bending stiffness of its beams and columns, the strength and rigidity of a panel structure is almost totally dependent upon the torsional rigidity of its plates. As might be expected, the analysis of panel furniture differs markedly from that of frame furniture. The objective of the latter is to determine joint rotations and member translations so that critical moments and consequent bending stresses in the beams and columns can be determined. Transverse and torsional shear forces are for the most part neglected. In panel furniture, the objective is reversed. Because of the great depth to width ratio of most plates, bending and transverse shear forces are seldom of concern. For the same geometric reason, determination of torsional shearing stresses and rotations is of primary concern. Although buckling and shear stress failures are important factors in the design of many plate type structures -- consider for example the torsional forces on an aircraft wing -- so that analysis of transverse and torsional shear forces is necessary, it is unlikely that shear stresses in plates used in furniture construction will ever approach critical stress levels. Thus the structural design of solid panel furniture can be reduced to an evaluation of torsional rotation and rigidity characteristics, or simply stated a torsional analysis.

Development of required analytical procedures begins with a recognition of the basic structure. When viewed from either the top or side, an open fronted, five-sided case may be regarded as a channel section with bulkheads attached to both ends. Torsional forces applied to either set of bulkheads will cause distortion of the open front. An analysis of the torsional resistance of this open channel section involves both a uniform and non-uniform torsional solution (Benjamin, 1958), and the total torsional rigidity is the sum of the two computations. Uniform torsion theory assumes pure shear flow around the section and bulkheads are assumed free to warp so that no bending stresses develop. Shear stress is constant across the wall and around the section so that the well known torsion-shear flow relationship, \( T = 2qA \), results (Shanley, 1944; Harris, 1959; Wang, 1953; Timoshenko and Goodier, 1951). Non-uniform torsion theory is more complex. If torsional forces are applied to the bulkheads which are assumed free to warp, the entire channel section will warp, and its rigidity is equal to the combined rigidity of each of its members considered separately. If warping of one bulkhead is restrained, the torsional stresses that arise at the warped end of the section are resolved into two oppositely directed shear forces in the sides or flanges of the section at the restrained end so that each flange now acts as a cantilever beam (Seeley, 1932; Seeley and Smith, 1952). The torsional rigidity of these panels acting as cantilever beams is of course much greater than when their ends are unrestrained, and the conclusion can immediately be drawn that stiffening one face of a case will greatly increase its torsional rigidity.

An exact solution of sections in which warping is restrained is quite complex and will not be considered here (Goldberg, 1953; Borg and Gennaro, 1959). Instead, a simplified analysis will be developed in which each panel is assumed attached to its neighbor at each corner by means of hinges. Forces can thus be transmitted from one panel to
Consider the open fronted, five-sided case shown in Figure 1. Should one support be removed the case will deflect downward under load at this point and each panel in the case will warp. Taken by itself, the torsional force, $T$, required to cause the bottom panel (B) to deflect downward 1 inch at support D (Figure 2) is given by the equation

$$T = \frac{ct^3G}{3a}$$

where

$$\frac{\Delta}{c} = \frac{Gt^3}{3a}$$

$T$ = applied torsional moment in pound inches

Assume in Figure 1, that $a = 36''$, $b = 48''$, and $c = 24''$ and that the bottom panel is $3/4$ inch thick yellow birch. At 12% M.C., $E_L$ for birch is $2.01 \times 10^6$ psi. Taking the average shear modulus as $1/16$ this value yields $G = 1.26 \times 10^5$ psi. Substituting these values into equation (1) and solving gives $T = 490$ inch pounds. According to the assumed boundary conditions, this torsional force will be resisted by an equal but opposite bending moment in the side panel, $(S_T)$. If $(S_T)$ is treated as a cantilever beam, an equivalent bending moment can be produced by applying a force of $P = 490/48 = 10.2$ pounds horizontally to the panel at point B in the plane of $(S_T)$. The deflection of a cantilever beam (Timoshenko and Young, 1945) due to bending stresses is given by

$$\Delta_M = \frac{PL^3}{3EI}$$

and due to transverse shearing stresses by
\[ \Delta V = \frac{1.2PL}{ctG} \] 

Assuming this panel were 1/4 inch thick
\[ \Delta_M = 0.00066'' \]

where \( I = \frac{tc^3}{12} = 288 \text{ in}^4 \) and
\[ \Delta_V = 0.00108'' \]

Therefore, considering only the bottom and right side panel, a force applied to the latter sufficient to cause the bottom panel to twist and deflect one inch will cause a combined bending and transverse shear deflection of 0.00174 inches in the side panel which is only 1/3 as thick. It appears reasonable, therefore, to neglect bending and transverse shear deflections in future calculations.

The deflection of point D due to uniform torsion effects may be calculated by treating the top, bottom, and sides as a closed section and assuming unrestrained bulkheads are attached to the front and back.

Utilizing shear flow concepts, the torsional moment acting on and in the plane of the front is given by

\[ T = 2\nu \alpha b \]

where \( \nu = \) the torsional unit shear stress. Because of these pure shear forces the panels will distort in their own plane, and panel \( (S_T) \) will rotate relative to panel \( (S_1) \). The distortion of \( (S_T) \) is given by

\[ \Delta_{S_T} = \frac{\nu c}{G} = \Delta_B = \frac{\nu c}{G} \] 

The rotation of \( (S_T) \) relative to \( (S_1) \) can be calculated using either faces \( (S_1) \) and \( (S_T) \) or \( (B) \) and \( (T) \) that is,

\[ \Theta = \frac{2\Delta}{a} \quad \text{or} \quad \Theta = \frac{2\Delta}{b} \]

but the true rotation is the average of the two:

\[ \Theta = \frac{\Delta}{a} + \frac{\Delta}{b} = \frac{Tc(a+b)}{2a^2b^2tG} \] 

(5)

Using the dimensions of the previous example,

\[ \Theta = \frac{Pac(a+b)}{2a^2b^2tG_{LT}} = 6.05P \times 10^{-8} \]

and \( \Delta_D = 12 \times 6.05 \text{ P} \times 10^{-8} = 7.26P \times 10^{-8} \) inches. For a 490 pound inch torque on panel (B) along its length, \( P \) must be 490/12 or 40.8 pounds. The deflection at D due to torsional shear stresses would thus be \( 2.96 \times 10^{-6} \) inches where the loading and support system is such as to cause a 1-inch deflection at D. Such a small contribution can be neglected with very little error. From the foregoing calculations, it can thus be concluded that only the torsional stiffness of the panels need be considered when evaluating the rigidity of a case since bending, transverse shear, and torsional shear deflections are small compared to deflections caused by warping. Let us now establish the relationship between the stiffness of the panels and the rigidity of the case.

If a flat panel such as B is supported on three corners and loaded on the fourth as shown in Figure 3, the rotation of line DH relative to CG will be given as previously discussed (Shanley, 1944; Michalos, 1958) by
Figure 4. A back panel (R) and side panel (S₁) have been added to the bottom panel (B) of Figure 3. Note that neither of these panels affects the deflection at point D, but the line EG is now fixed in space.

the expression
\[ \Theta = \frac{3TL}{ct^{3}G} \]  (6)

If panels (S₁) and (R) are now added (Figure 4) to panel (B), they will add nothing to the rigidity of the panel, but note that line EG has been fixed in the space so that E is not free to move right or forward. Furthermore, any rotation of the structure must take place around line EG. If panel (Sᵢ) is added (Figure 5) it will connect with the back and bottom panels (R) and (B) so that for a deflection \( \Delta_D \), point F will deflect forward along line BF an amount

\[ \Delta_F = \frac{b}{c} \Delta_D \]  (7)

Angle \( \psi \) which measures the rotation of the back, that is, the relative rotation of line

Figure 5. Diagram showing the geometrical relationships existing between the various panels for a given deflection at D when a side panel (Sᵢ) is added to Figure 4.

EF to GH is given by
\[ \psi = \frac{\Delta_f}{a} = \frac{b}{a} \quad \rho = \frac{b}{a} \quad \Theta \]  (8)

Note that the rotation of the back could just as well be described by the rotation of FH relative to EG. If the top panel, T, is now added (Figure 6) it must deflect downward an amount \( \Delta_D \), but it will also cause points B and A to move to the left along line AB an amount

\[ \Delta_A = \frac{b}{a} \Delta_D \]  (9)

In other words, plate T rotates about point E and members (S₁) and (Sᵢ) rotate about their axes by an angle \( \psi \) just as the back. These relationships are further clarified in the two-dimensional views of Figure 7.
Figure 6. This diagram shows the relationship between the deflections of each panel of the completed five-sided case and the deflection at D.

The relationship of the forces transmitted from one panel to the next must also be established. These are derived using principles of statics. As can be seen in Figures 6 and 7, a load, $P_1$, acting downward at D causes a force $P_2$ to act on panel (R) along line BF. Similarly, it causes a load $P_3$ to act on panels (S) and (S') along line AB. The relationship of $P_2$ and $P_3$ to $P_1$ is given by

$$P_2 = \frac{c}{b} \cdot P_1$$

and

$$P_3 = \frac{a}{c} \cdot P_2 = \frac{a}{b} \cdot P_1 \quad (10)$$

Now, the torsional rigidity of each panel is given by

$$\frac{T}{\Theta} = \frac{ct^3G}{3L}$$

What must next be done is to establish a relationship between angle rotations, forces,

Figure 7. Top, front, and side views of the case shown in Figure 6 which further show the deflection and rotational relationships of panels in the case.

and torsional stiffness of each panel and the stiffness of the case.

Considering the bottom first

$$\frac{T_B}{\Theta} = \frac{P_1c}{\Theta} = \frac{3}{ct_BG_B}$$

or

$$\frac{P_1}{\Theta} = \frac{1}{c} \cdot \frac{ct_BG_B}{3a} \quad (11)$$

The load-rotation characteristics of the top will be similar to that of the bottom

$$\frac{T_T}{\Theta} = \frac{3}{ct_TG_T}$$

or

$$\frac{P_1}{\Theta} = \frac{1}{c} \cdot \frac{ct_TG_T}{3a} \quad (12)$$
The relationship of the back to $P_1$ and $\Theta$ should be independent of whether we consider angle $\psi$ or $\phi$. Treating $\psi$ first gives

$$\frac{T_R}{\psi} = \frac{P_2 a}{b} \frac{\Theta}{a} = \frac{a b P_1}{b} \frac{\Theta}{a} = \frac{a t^3 G_R}{3b}$$

or

$$\frac{P_1}{\Theta} = \frac{b^2}{a^2 c} \frac{a t^3 G_R}{3b} = \frac{b t^3 G_R}{3a}$$

(13)

Considering the torque of $R$ from the side

$$\frac{T_R}{\Theta} = \frac{P_2 b}{\Theta} = \frac{c P_1}{\Theta} = \frac{b t^3 G_R}{3a}$$

or

$$\frac{P_1}{\Theta} = \frac{1}{c} \frac{b t^3 G_R}{3a}$$

(14)

The relationship of the twisting of the sides to $P_1$ and $\Theta$, assuming sides $(S_1)$ and $(S_r)$ are identical, is given by

$$\frac{T_S}{\psi} = \frac{P_3 c}{b} \frac{\Theta}{a} = \frac{P_1}{\Theta} \frac{a^2 c}{b^2} = \frac{c t^3 G_S}{3b}$$

or

$$\frac{P_1}{\Theta} = \frac{b^2}{a^2 c} \frac{c t^3 G_S}{3b}$$

(15)

Summing the force rigidity expressions for each rectangular element gives the following relationship for the load rigidity of the entire case

$$\frac{P_1}{\Theta} = \frac{1}{c} \frac{c t^3 G_B}{3a} + \frac{1}{c} \frac{c t^3 G_T}{3a} + \frac{2b^2}{a^2 c} \frac{c t^3 G_S}{3b} + \frac{b^2}{a^2 c} \frac{c t^3 G_R}{3b}$$

(16)

which may be simplified to

$$\frac{P_1}{\Theta} = \frac{t^3 G_B}{3a} + \frac{t^3 G_T}{3a} + \frac{2b t^3 G_S}{3a^2} + \frac{b t^3 G_R}{3ac} = V$$

(17)
Finally, the relationship of the sum of the stiffness of the panels to the deflection of joint D under load $P_1$ is given by

$$\Delta_D = \frac{P_1 c}{V} \quad (18)$$

Although the above expression is quite useful when applied to flat panels made of plywood, hardboard, or solid wood, it is difficult to apply to framed panels since they are in themselves composite structures and equivalent t and G values must be assumed for each panel. The arbitrary character of these assumptions would likely make a rigidity analysis meaningless when applied to a real case. Rather than determine equivalent values for t and G, and attempting to use the above expression to evaluate the deflection characteristics of a case, it appears feasible to determine the stiffness of each panel experimentally and then determine the deflection characteristics of the case from a consideration of the stiffness of each panel.

Referring to equation 16, the torsional rigidity of each panel is given by the appropriate quantity in parentheses. Rewriting this expression gives

$$\frac{P_1 c}{D} = \frac{1}{c} \cdot \frac{T_B}{\Theta_B} + \frac{1}{c} \cdot \frac{T_T}{\Theta_T} + \frac{2b^2}{a^2 c} \cdot \frac{T_S}{\Theta_S} + \frac{b^2}{a^2 c} \cdot \frac{T_R}{\Theta_R} \quad (19)$$

where $\Theta$ in this case refers to the twist of each panel considered separately. For panels supported at three corners and a load applied to the fourth, this expression can be written as

$$\frac{P_1 c}{\Delta_D} = \frac{1}{c} \cdot \frac{P_B}{\Delta_B} c^2 + \frac{1}{c} \cdot \frac{P_T}{\Delta_T} c^2 + \frac{2b^2}{a^2 c} \cdot \frac{P_S}{\Delta_S} c^2 + \frac{b^2}{a^2 c} \cdot \frac{P_R}{\Delta_R} c^2 \quad (20)$$

Solving for $\Delta_D$ yields

$$\Delta_D = \frac{P_1 c}{\left( \frac{P_B}{\Delta_B} + \frac{P_T}{\Delta_T} + \frac{2b^2}{a^2 c} \cdot \frac{P_S}{\Delta_S} + \frac{b^2}{a^2 c} \cdot \frac{P_R}{\Delta_R} \right)} \quad (21)$$

If, in testing panel rigidity, the same load, say $P_B$, is applied to each panel, the rigidity of the case may be expressed as follows

$$\Delta_D = \frac{P_1 c}{1 + \left( \frac{\Delta_B}{\Delta_T} + \frac{2b^2}{a^2 c} \cdot \frac{\Delta_B}{\Delta_S} + \frac{b^2}{c^2} \cdot \frac{\Delta_B}{\Delta_R} \right)} \quad (22)$$

The value of the above expression lies in the fact that the stiffness of the entire structure can be predicted by simply testing the stiffness of each panel separately and substituting its relative stiffness into the above expression.
Figure 8. Drawing of the small test case analyzed in this paper.

To demonstrate the application of the above analytical procedure two example cases will now be analyzed. Deflection data from a previous test is first used to demonstrate how well the deflection of one edge of a small drawer-like structure agreed with that predicted theoretically. Dimensions of the test structure (Figure 8) were 11-7/8 inches long, 9-1/4 inches wide and 6 inches deep. A 1/2 x 1/2 x 6 inch solid wood block was glued in each corner which joined the sides together. The bottom was then attached at each corner to the end of one of the blocks by a wood screw. All panels were 1/8 inch thick Masonite. The torsional rigidity of the back was determined experimentally and the relative stiffness of the other panels then calculated in terms of it. Thus, when the case is placed so that the 6 x 11-7/8 inch panels form the top and bottom and the 6 x 9 inch panels the sides.

\[
\frac{T}{\Theta} \bigg|_S = \frac{9.14}{6} \times \frac{11.78}{8} \times \frac{T}{\Theta} \bigg|_R = 2.32 \times \frac{T}{\Theta} \bigg|_R
\]

\[
\frac{T}{\Theta} \bigg|_B = \frac{9.14}{6} \times \frac{11.78}{10.58} \times \frac{T}{\Theta} \bigg|_R = 1.73 \times \frac{T}{\Theta} \bigg|_R
\]
Differences from the 6, 9-1/4 and 11-7/8 inch dimensions occur because the 11-7/8 x 9-1/4 inch Masonite on the ends overlapped the sides and the width of the corner blocks must be subtracted from the length of any panel to which it is attached. Since the numerical rigidity of only one panel was determined, the predicted deflection of a corner could best be determined from equation 19. Dividing by $\Theta_R$ and substituting relative stiffness gives

$$\frac{P_{1C}}{\Delta} = \frac{2}{c} \cdot (1.73) + \frac{2}{c} \cdot \frac{b}{a^2} \cdot (2.32) + \frac{1}{c} \cdot (1)$$

Note that expression (14) was used for the back rather than (13).

Substituting $T_R = P_{1C}, \Theta = \frac{\Delta_R}{b}$ for this edge, and simplifying the above expression gives

$$\frac{\Delta_R}{\Delta_D} = 2 \cdot (1.73) + \frac{2b^2}{a^2} \cdot (2.32) + 1 \cdot \frac{b^2}{c^2}$$

$$\Delta_R = (7.25 \times 2.25) \quad \Delta_D = 16.2 \quad \Delta_D$$

or

$$\Delta_D = \frac{\Delta_R}{16.2}$$

inches where $b = 9-1/4$ inches and $a = 11-7/8$ inches. Deflection of the back panel originally was about 0.370 inches. Based on this value, deflection of the case under an equal load at a point corresponding to support D in the original drawing should be

$$\Delta_D = 0.370/16.2 = .0228 \text{ inches}$$

The measured value was 0.025 inches, or the actual was 9.6 percent more than predicted from measurements made on the back panel alone.

An additional example illustrates the applicability of the analytical procedure to larger cases. Willard (1964) published deflection data for a 42 x 42 x 20 inch deep case constructed of 3/16 inch thick plywood or Masonite bottom, sides, and back, and a 1-inch thick, 5-ply lumber core plywood top. Although his support and loading systems were somewhat different from those described in this paper, the results should be applicable because of the great stiffness of the panels in their own plane. A torsional rigidity value for a 13/16-inch lumber core panel was given from which a value for the modulus of rigidity of the top panel was calculated. Similar values could not be determined for the remaining panels, but since they contributed much less to the rigidity of the case than the top panel, it was felt that a rigidity
modulus equal to about 1/16 the modulus of elasticity of Douglas fir could be safely assumed (Anon. 1, 1951).

Published and derived values for the case are as follows:
\[
\begin{align*}
  a &= b = 42 \text{ inches} \\
  c &= 20 \text{ inches} \\
  G_T &= 192,000 \text{ psi} \\
  G \text{ (bottom, back, sides)} &= 100,000 \text{ psi} \\
  t_T &= 1 \text{ inch} \\
  t_B = t_{sl} = t_{sr} = t_R &= 3/16 = 1/8 \text{ inch}
\end{align*}
\]

Substituting these values into equation (18) and taking into account the differences in loading and support systems yields the following result:
\[
\frac{20P_1}{\Delta} = \left( \frac{27}{4100} \times \frac{100,000}{126} \right) \times 2 + \left( \frac{192,000}{126} \right) + \left( \frac{27}{4100} \times \frac{100,000}{60} \right)
\]

or, \( P_1 = 9.65# \)

as compared to the published experimental value of 8.5 pounds. Considering the number of assumptions made, the predicted load is remarkably close to the experimentally found load; but because of the assumptions made, too much should not be read into the results. It is interesting to examine the contribution of each panel to the stiffness of the case. The top 1-inch panel provided 98.2\% of the stiffness whereas each side provided 0.36\% and the back, 0.7\%. This demonstrates, as was shown previously and as Willard and others (Anon. 2, 1958) have pointed out, the value of stiffening at least one face of the case.

**SUMMARY**

Despite numerous superficial differences most case and carcass furniture is made up of only three structural elements -- beams, columns, and plates -- which form two basic components: a rigid frame and solid or framed panels. The analysis of furniture in which a rigid frame is the predominant structural component has been discussed in previous papers (Eckelman, 1966b, c). In this paper the analysis of paneled furniture made up of solid and/or framed plates is discussed. The analysis of such furniture involves a consideration of the torsional rigidity of the panels rather than an analysis of transverse shear and bending stresses as in framed structures. Using the appropriate geometric relationships established above, the rigidity of a case can be determined by an analysis of the rigidity of each of its panels. The practical group of furniture which can be theoretically analyzed by this method is limited to solid panel furniture or to framed panel furniture in which the rigidity of each panel can be determined experimentally. Nevertheless, by giving an insight into the structural behavior of this type of
furniture, the analytical procedure gives a firm foundation for analyzing and understanding various measures which have been proposed for stiffening case and carcass furniture.

LITERATURE CITED


