Effect of Holes on the Bending Strength of Wood and Particleboard Parts

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When constructing furniture frames, it is ordinarily necessary to bore holes through the members to join them together by means of fasteners such as dowels and screws. These holes must be expected to have a weakening effect upon the members, particularly if they occur at points of high stress. To insure that members are adequately designed to resist the forces which will be imposed on them in service, it is important that any such reductions in strength be taken into account in the design process.

Holes may reduce the strength of a member in two ways. First, they diminish the amount of material in the cross section of a member so that a loss of strength may be expected, which is in some way related to the amount of material removed. Secondly, they may give rise to stresses around their periphery which are several times greater than the average stress prevailing in the region (stress concentrations) and thus have a greater weakening effect than would be predicted by calculations based only on the amount of material removed from the cross section of the part. Whether such stress concentrations affect the ultimate strength of wood specimens is subject to question. Wood has plastic properties so that local stress concentrations might be expected to be relieved by relaxation of the material. If this supposition is true, the weakening effect of holes can then be calculated on the basis of loss in cross section, residual strength can easily be determined, and overall design problems held to a minimum. If stress concentrations must be taken into account, however, the design problem becomes much more complex. It is important, therefore, to resolve whether stress concentration factors must be taken into account in the design of furniture parts with holes in them.

In this paper, the expressions are developed and presented which are required to calculate the residual strength of members with holes in them. The results of tests which were carried out to evaluate these expressions as they apply to wood and particleboard parts are also presented. Development of the desired residual strength formulas is given first.

**Calculation of Residual Strength**

To calculate the reduction in strength which occurs when a hole is drilled through or into a member on the basis of net remaining cross section, it is necessary to take into account both the change in neutral axis and the change in moment of inertia that occurs in the beam owing to removal of material for the hole. The modulus of rupture (MOR) for a hole-free beam is given by

\[ \text{MOR} = \frac{M\dot{\gamma}}{I} \]

where \( M \) = the ultimate applied moment
\( \dot{\gamma} \) = the distance from the neutral axis to the extreme fiber
\( I \) = the moment of inertia of the beam.

If we assume that no stress concentrations are present around the hole, then the modulus of rupture (MOR\(_h\)) of a beam with a hole in it may be found by means of a similar expression; namely

\[ \text{MOR}_h = \frac{M_h\dot{\gamma}_h}{I_h} \]

where \( M_h \) = ultimate moment applied to the beam
\( \dot{\gamma}_h \) = distance from the neutral axis to extreme fiber of the beam
\( I_h \) = moment of inertia of the beam about the neutral axis.

The method of attack employed here is to assume that the modulus of rupture of a beam with a hole in it is numerically equal to the modulus of rupture of a hole-free beam; i.e.,

\[ \text{MOR}_h = \text{MOR} \]

and then to prove or disprove this assumption by experiment. Specifically, if stress concentrations can be ignored for a beam subjected to an ultimate load, then the expected value of the strength ratio will be given by

\[ \frac{M_h}{M} = \frac{I_h\dot{\gamma}_h}{I\dot{\gamma}} \]

Thus the ratio of the bending force which the piece with the hole can carry compared to a piece of solid cross section, that is, the percentage of strength retained by the member with the hole in it, is directly proportional to the ratio of the moments of inertia of the two sections and inversely proportional to the ratio of the corresponding extreme fiber distances from their respective neutral axes.

To determine the percentage of remaining strength in a beam with a hole in it, therefore, it is necessary to calculate \( I_h, I, \dot{\gamma}_h, \) and \( \dot{\gamma} \), substitute these values into equation (3), and solve the
equation. In the case of rectangular beams with which we are concerned in this study,

\[ I = \frac{bh^3}{12} \]

about the neutral axis of the beam, and

\[ \bar{y} = \frac{h}{2} \]

where \( b \) = the width of the part as defined in Figures (1) through (7)

\( h \) = the height of the member

Only \( I_{II} \) and \( \bar{y}_{II} \) remain to be determined to calculate the strength ratio.

In evaluating these two quantities there are essentially only two principal cases to consider. The first case occurs when a hole is drilled through a member parallel to the line of action of an applied load; the second case occurs when a hole is drilled through a member perpendicular to the axis of the load. Variations of these two cases are common. In some cases, for example, the holes may not be drilled all the way through the member. These two cases along with two variations may be categorized as follows.

**Case 1a.** (Member laid flat, hole drilled through parallel to line of action of load.)

When a hole is drilled through a piece parallel to the line of action of an applied load as shown in Figure 1a, \( \bar{y}_{II} \) remains equal to \( h/2 \) just as for a solid beam; the new moment of inertia may be calculated as

\[ I_{II} = \frac{bh^3}{12} - \frac{Dh^3}{12} - \frac{(b-D)h^3}{12} \]  

(5)

where \( D \) = the diameter of the hole. In other words, for this special case, the loss in moment of inertia, and consequently, the loss in bending strength is directly proportional to the diameter of the hole; i.e., the percentage of remaining cross section. This statement also holds if more than one hole is drilled through the member at a given section, Figure 1b; in this case, \( D \) is equal to the sum of the diameter of the holes.

**Case 1b.** (Member on edge, hole drilled partially through beam parallel to line of action of load on bottom side of beam.)

If a hole is not drilled all the way through a member, however, the calculations become more complex. This situation is illustrated in Figure 2, where a cross section of such a member is shown. To find the neutral axis we make use of the principle that the moment of any area about a point is equal to the algebraic sum of the moments of its component areas. Thus, as shown in Figure 3, if we take moments about the bottom of the cross section we obtain

\[ (bh-Dd)\bar{y}_{II} = bh^2/2 - Dd^2/2 \]

(6)

or,

\[ \bar{y}_{II} = \frac{bh^2 - Dd^2}{2(bh - Dd)} \]

(7)

where \( D \) = diameter of hole

\( d \) = depth of hole

Once the neutral axis has been located, then the moment of inertia, \( I_{II} \), of the member with respect to the neutral axis can be found, Figure 3, by means of the expression

\[ I_{II} = \frac{bh^3}{12} + bh(\bar{y}_{II}-h/2)^2 \]

\[ - \frac{Dh^3}{12} - Dd(\bar{y}_{II}-d/2)^2 \]

(8)

**Figure 2.** A specimen to be tested on edge with a hole drilled partially through parallel to line of action of load. (Case 1b in text; and specimen groups (5) through (8), Table 3.)
Figure 3. Derivation of expressions needed to find neutral axis and moment of inertia of a beam with a hole drilled partially through it on bottom side parallel to the line of action of the load, Case 1b.

Case 2a. (Member on edge, hole drilled through, perpendicular to line of action of Load.)

When a hole is drilled through a piece perpendicular to the line of action of an applied load as shown in Figure 4, a new set of expressions is needed to find the neutral axis and moment of inertia. Referring to Figure 5, if we again take moments about the bottom of the cross section, we obtain

\[(bh - bD)\bar{y}_H = bh \cdot \frac{h}{2} - bD \cdot w = \frac{b}{2} (h^2 - 2Dw)\]  

where \(w\) = the distance from the bottom of the piece to the center of the hole. Rearranging terms gives

\[\bar{y}_H = \frac{h^2 - 2Dw}{2(h - D)}\]  

Finally, solving for \(I_H\) with respect to the neutral axis gives

\[I_H = \frac{bh^3}{12} + bh (\frac{h}{2} - \bar{y}_H)^2 - \frac{bD^3}{12} - bD (w - \bar{y}_H)^2\]  

Figure 4. A specimen to be tested on edge with hole drilled through perpendicular to line of action of applied load (Case 2a; groups (9) through (14), Table 4, and groups (15) through (17), Table 5).
Figure 6. A specimen to be tested in the flat position with hole drilled partially through perpendicular to line of action of applied load. (Case 2b; group (18), Table 6).

Case 2b. (Member laid flat, hole drilled partially through, perpendicular to line of action of the load.)

When a hole is drilled into a piece perpendicular to the line of action of the load but does not pass completely through the part as shown in Figure 6, the neutral axis must then be found, Figure 7, as

\[(bh - aD)\bar{y}_H = bh(h/2) - aDw\]  

(12)

where \(a\) = depth of penetration of the hole in the member.

Solving for \(\bar{y}_H\) gives

\[\bar{y}_H = \frac{bh^2 - 2aDw}{2(bh - aD)}\]  

(13)

Calculating the new moment of inertia with respect to this neutral axis gives

\[
I_H = \frac{bh^3}{12} + bh(\bar{y}_H - h/2)^2 - \frac{aD^3}{12} - aD(\bar{y}_H - w)^2
\]  

(14)

Figure 7. Derivation of equations needed to calculate neutral axis and moment of inertia of beam with hole drilled partially through it perpendicular to line of action of applied load (Case 2b; group (18), Table 6).

Test Procedures and Results

Static bending tests were carried out on both wood and particleboard beams of rectangular cross section to evaluate the predictive power of the expressions which had been developed. Throughout these tests matched control specimens were used as shown in Figure 8. One specimen of a matched pair had a hole drilled into it, whereas the remaining specimen did not. The strength of the beam with a hole in it could thereby be expressed as a percentage of the solid control specimen.

The cross sectional geometry of each set of specimens along with hole size and location are given in Tables 1 through 6. Each member was tested as a beam, either on edge or laid flat, subjected to center point loading as shown in either Figure 1 or 2. Matched control specimens were needed because of the considerable variability in strength of both wood and particleboard. Even with matched specimens, however, considerable variability was still noted.

Results of the tests are given in Tables (1) through (6). The strength values obtained for beams with holes in them were first expressed as a percentage of the values obtained for the
Table 1. Bending strength results obtained with Douglas-fir specimens tested in flatwise bending position. Case 1a. Holes were drilled through the specimens parallel to the axis of the load, Figure 1.

<table>
<thead>
<tr>
<th>Specimen group</th>
<th>No. of specimens</th>
<th>Species</th>
<th>Beam width</th>
<th>Hole diameter</th>
<th>Percentage remaining X-section</th>
<th>Strength ratios</th>
<th>Observed/predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inches</td>
<td>Inches</td>
<td></td>
<td>Observed</td>
<td>Predicted</td>
</tr>
<tr>
<td>1 (one hole)</td>
<td>10</td>
<td>D-fir</td>
<td>1.585</td>
<td>0.25</td>
<td>82.4</td>
<td>84.2</td>
<td>82.4</td>
</tr>
<tr>
<td>2 (two holes)</td>
<td>5</td>
<td>Mixed</td>
<td>1.532</td>
<td>0.75</td>
<td>51.0</td>
<td>55.1</td>
<td>51.0</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>1.537</td>
<td>.375</td>
<td>51.2</td>
<td>52.3</td>
<td>51.2</td>
</tr>
</tbody>
</table>

The corresponding control specimens. These ratios are given in the tables under the heading “Strength Ratios—Observed.” The strength of the beam with a hole in it along with the strength of its corresponding control were next calculated on a theoretical basis. The strength ratios of these theoretically obtainable values were then calculated. These ratios are given in the tables under the heading “Strength Ratios—Predicted.” Finally, the “observed” strength ratios were compared to the “predicted” strength ratios and expressed as a percentage; i.e., (Observed Ratio/Predicted Ratio) x 100 percent. These ratios are given under the heading “Observed/Predicted.” To evaluate the results it is convenient to separate the data into the same categories that were previously used in developing the various expressions.

Case 1a.

The results of these tests are given in Tables (1) and (2). In the case of solid wood with a single hole drilled through it, specimen group (1), the average ratio was 102.6 percent; i.e., the predicted values were very slightly lower than observed values. Such close agreement, however, tends to indicate that the strength of such members can in fact be calculated on the basis of remaining cross section. Furthermore, the results of tests with two holes drilled side by side, Figure 1b, group (2), indicate that the effect of drilling more than one hole side by side is simply additive. The ratio obtained in this case was 102.1 percent, which again represents close agreement with predicted results.

Results obtained with particleboard are less clear. In one case, group (3), the observed/predicted ratio was only 87.1 percent. The average ratio for the remaining cases, group (4), however, was 98.5 percent which again indicates a close agreement. The low values observed indicate that some caution should be exercised in calculating strength on the basis of remaining cross section, particularly in those cases where the holes occupy a rather high percentage of the cross section.

Case 1b.

The results obtained with the first group of specimens in these tests, group (5), Table 3, were interesting in that essentially the same results were obtained regardless of whether the holes

Table 2. Bending strength results obtained with hardwood particleboard specimens tested in the flatwise bending position, Case 1a. Holes were drilled through the specimens parallel to the axis of the load, Figure 1.

<table>
<thead>
<tr>
<th>Specimen group</th>
<th>No. of specimens</th>
<th>Beam width</th>
<th>Hole diameter</th>
<th>Percentage remaining X-section</th>
<th>Strength ratios</th>
<th>Observed/predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inches</td>
<td>Inches</td>
<td></td>
<td>Observed</td>
<td>Predicted</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>2.830</td>
<td>0.75</td>
<td>73.5</td>
<td>64</td>
<td>73.5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.755</td>
<td>0.75</td>
<td>57.2</td>
<td>54.0</td>
<td>57.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.000</td>
<td>0.75</td>
<td>42.8</td>
<td>39.7</td>
<td>42.8</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.625</td>
<td>0.75</td>
<td>64.2</td>
<td>61.5</td>
<td>64.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.500</td>
<td>0.75</td>
<td>72.6</td>
<td>71.4</td>
<td>72.6</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.375</td>
<td>0.75</td>
<td>81.6</td>
<td>79.1</td>
<td>81.6</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.250</td>
<td>0.75</td>
<td>85.5</td>
<td>85.5</td>
<td>85.5</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean (group 4) = 98.5
were drilled to 20, 40, or 60 percent of the depth of the beams. Theory, however, predicts only about a 13-percent change in strength from least to greatest depth so that these results could in fact be due to chance alone.

Subsequent tests, groups (6), (7), and (8) produced somewhat mixed results. The weighted mean for all of the samples in this group, however, was 108.6 percent which indicates that on the average, net remaining section can be used to obtain reasonable estimates of remaining strength.

Case 2a.

Results of tests carried out on a softwood, Douglas-fir, and various hardwoods are given in Table 4. In the case of the softwood samples, group (9), the holes were drilled through the exact center of each piece. The strength values obtained for these particular specimens were somewhat lower than was predicted on the basis of net remaining section, particularly as the size of the holes increased. It could be seen, however, that these specimens were failing in compression which presumably contributed to the low values noted, especially for those specimens with large holes relative to the size of the specimen. The average failing ratio for this set of 20 matched specimens was 96.4 percent, but it must be recognized that the high value obtained for the one subset of five specimens helped to offset the much lower values noted for the other three subsets.

The holes in the hardwood samples were drilled off-center; that is, much nearer the edge of the beam. The weighted average strength ratio noted for groups (10) through (14) was 106.5 percent,

Table 4. Bending strength results obtained with various softwood and hardwood specimens tested in the edge position. Holes were drilled through the specimens perpendicular to the line of action of the load, Case 2a, Figure 4.

<table>
<thead>
<tr>
<th>Specimen group</th>
<th>No. of specimens</th>
<th>Species</th>
<th>Beam width</th>
<th>Hole diameter</th>
<th>Hole center to edge</th>
<th>Strength ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Inches</td>
<td></td>
<td></td>
<td>Observed</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>Doug-fir</td>
<td>1.531</td>
<td>0.375</td>
<td>0.766</td>
<td>0.578</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td></td>
<td>1.532</td>
<td>0.500</td>
<td>0.766</td>
<td>0.516</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td></td>
<td>1.528</td>
<td>0.625</td>
<td>0.764</td>
<td>0.452</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td></td>
<td>1.533</td>
<td>0.750</td>
<td>0.767</td>
<td>0.392</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>89.8</td>
</tr>
</tbody>
</table>

Mean (group 9) = 96.4

10              | 8                | White Oak  | 1.950      | 0.375         | 0.538               | 0.350   | 885.1     | 76.2     | 111.7     |
| 11             | 13               | Red Oak    | 3.15       | 0.375         | 0.441               | 0.253   | 75.5      | 71.9     | 105.0     |
| 12             | 8                | Red Oak    | 3.043      | 0.500         | 0.676               | 0.376   | 72.0      | 71.0     | 102.7     |
| 13             | 15               | Red Oak    | 3.141      | 0.500         | 0.626               | 0.376   | 72.0      | 71.0     | 102.7     |
| 14             | 10               | Yel Poplar | 3.159      | 0.125         | 0.318               | 0.255   | 89.2      | 106.1    |
| Total          | 54               |            |            |               |                     | 885.1   | 76.2      | 111.7     |

Mean (groups 10-14) = 106.5
Table 5. Bending strength results obtained with hardwood particleboard tested in the edge position. Holes were drilled through the specimens perpendicular to the line of action of the load, Case 2a, Figure 4.

<table>
<thead>
<tr>
<th>Specimen group</th>
<th>No. of specimens</th>
<th>Beam width</th>
<th>Hole diameter</th>
<th>Hole center to edge</th>
<th>Edge thickness</th>
<th>Strength ratio</th>
<th>Observed</th>
<th>Predicted</th>
<th>Observed/predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>3</td>
<td>3.919</td>
<td>.4375</td>
<td>.46</td>
<td>.237</td>
<td>57.7</td>
<td>70.8</td>
<td>81.5</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>3.921</td>
<td>.4375</td>
<td>.83</td>
<td>.610</td>
<td>73.7</td>
<td>81.4</td>
<td>90.5</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>3.920</td>
<td>.4375</td>
<td>1.19</td>
<td>.973</td>
<td>96.3</td>
<td>89.7</td>
<td>107.4</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>3.912</td>
<td>.4375</td>
<td>1.59</td>
<td>1.370</td>
<td>95.3</td>
<td>96.2</td>
<td>99.1</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>6</td>
<td>3.968</td>
<td>.4375</td>
<td>1.92</td>
<td>1.700</td>
<td>103.3</td>
<td>99.4</td>
<td>104.9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>6</td>
<td>3.983</td>
<td>.4375</td>
<td>1.258</td>
<td>1.038</td>
<td>97.9</td>
<td>90.7</td>
<td>107.9</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>4.018</td>
<td>.4375</td>
<td>0.519</td>
<td>0.299</td>
<td>91.9</td>
<td>73.1</td>
<td>84.7</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>4.018</td>
<td>.4375</td>
<td>.619</td>
<td>0.569</td>
<td>76.6</td>
<td>80.4</td>
<td>95.3</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>4.015</td>
<td>.4375</td>
<td>1.674</td>
<td>1.454</td>
<td>91.5</td>
<td>96.9</td>
<td>94.4</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>4.001</td>
<td>.4375</td>
<td>2.013</td>
<td>1.793</td>
<td>99.2</td>
<td>99.9</td>
<td>99.3</td>
<td></td>
</tr>
</tbody>
</table>

Total 33

Mean (groups 15-17) = 97.5

6.5 percent greater than predicted. Again, this appears to be a reasonably close agreement between predicted and observed values.

Results obtained with particleboard samples, Table 5, were somewhat lower than predicted, particularly when the holes were drilled close to the edge of the piece, which again raises the possibility that stress concentrations may exist around holes in particleboard. The overall weighted average noted, 97.5 percent (groups 15, 16 and 17) indicates good agreement between theory and practice, but the low values obtained for holes drilled from 1/4 to 1/2 inch from the edge raise doubts. Because of the variability of the material, additional tests with larger numbers of samples are needed to resolve the issue.

Case 2b.

Values obtained in these tests are given in Table 6. In general, observed and predicted strength ratios agree relatively closely. Theoretically, the loss in strength suffered by a member with holes drilled into it as in this test, however, is so small that a large number of tests would be required to determine exactly how well the values actually agree. The primary purpose of this test, therefore, was to investigate the possibility that a reduction in strength might occur which would be substantially greater than expected owing to some unforeseen circumstance. The conservative results obtained tend to rule out such possibility.

Conclusions

In general, the results of these tests indicate that calculations based on net remaining section provide reasonable estimates of the strengths of wood parts after holes are drilled into them. When holes were drilled near the edge of particleboard beams tested on edge, however, observed results dropped nearly 15 to 20 percent below predicted values. This result indicates that stress concentrations may occur around holes in particleboard beams. Additional tests are needed, however, to further evaluate the magnitude and conditions of occurrence of this effect.

Table 6. Bending strength results obtained with tupelo specimens tested in the flat position. Holes were drilled partially through the specimen perpendicular to the line of action of the load, Case 2b, Figure 6.

<table>
<thead>
<tr>
<th>Specimen group</th>
<th>No. of specimens</th>
<th>Species</th>
<th>Beam width</th>
<th>Beam thickness</th>
<th>Hole diameter</th>
<th>Hole depth</th>
<th>Strength ratio</th>
<th>Observed</th>
<th>Predicted</th>
<th>Observed/predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>9</td>
<td>Tupelo</td>
<td>3.03</td>
<td>0.853</td>
<td>0.375</td>
<td>1.466</td>
<td>97.2</td>
<td>95.9</td>
<td>101.4</td>
<td></td>
</tr>
</tbody>
</table>

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